FIRST FUNDAMENTAL PROBLEM FOR A PIECEWISE-HOMOGENEOUS PLANE WITH A SLIT PERPENDICULAR TO THE LINE OF SEPARATION

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The complete solution for arbitrary boundary conditions in the stresses on the edges of a slit is constructed from the particular solution on the effect of a given difference in the stresses, and from the general solution of the problem on the effect of given stresses of equal magnitude on the edges of the slit. This latter is equivalent to two different problems for a composite half-plane (two rectangular wedges with different elastic characteristics) with mixed boundary conditions, which reduces to the same Wiener-Hopf equation solved by means of factorization of the kernel [1].

The solution for a loading of the form r^m ($m \geqslant 0$ an integer) is obtained in closed form, and when necessary is easily generalized for a polynomial or power series loading with a radius of convergence exceeding the length of the slit. Expressions are presented for the stress intensity coefficients ($m = 0$—3), as well as for the asymptotic values ($m \geqslant 4$).

1. Formulation of the problem. The solution of the problem for a wedge with a slit on the bisectrix when the stresses applied to the edges of the slit are equal is given in [2] by reducing the pair of integral equations to a Fredholm integral equation, and in [3] for the case of arbitrary values of the stresses on the slit edges by utilizing the Wiener-Hopf functional equation with the solution of the conjugate problem written in general form.

A half-plane with a notch perpendicular to the boundary was considered in [4 – 7]. A functional Wiener-Hopf equation has been obtained in [4] for the given problem and the magnitude of the energy released has been calculated for a slit in a field of eccentric tension; the case of simple tension was considered in [5 and 6], and a general solution is given in [7] for the problem with preliminary extraction of a particular solution of the corresponding difference in the stresses on the slit edges.

Let the domain occupied by the elastic body be a piecewise-homogeneous plane S with a slit along the segment [0, 1] of the Ox-axis (Fig. 1). The elastic constants of the right and left half-planes $S_{1,2}$ are denoted, respectively, by $(\mu, \varkappa; E, \nu)_{1,2}$. We define the boundary stresses at point $y = \pm 0$ of the slit edges by equalities

$$(Y_y - iX_y)_+ (t) = s(t) + q(t) \qquad (Y_y - iX_y)_- (t) = s(t) - q(t) \qquad (1.1)$$

where $s(t)$, $q(t)$ are bounded complex functions for which

$$q(0) = q(1) = 0$$

Let us construct some particular solution of the problem with zero stresses at infinity which satisfies the condition

$$(Y_y - iX_y)_+ (t) - (Y_y - iX_y)_-(t) = 2q(t) \tag{1.2}$$

at points of the slit.

For the homogeneous domain S the considered particular solution of the problem can be taken as ([8], p. 441) (1.3)

$$[\Phi, \Psi]^* (z) = \frac{1}{2\pi i} \int_0^1 [\Phi, \Psi]^* (z, t)\, dt, \quad \Phi^* (z, t) = \frac{q(t)}{t-z}, \quad \Psi^* (z, t) = \frac{\overline{q(t)}}{t-z} - \frac{tq(t)}{(t-z)^2}$$

The equilibrium and continuity conditions on the line $x = 0$

$$\Phi(+0, y) + \overline{(\Phi - z\Phi' - \Psi)(+0, y)} = \Phi(-0, y) + \overline{(\Phi - z\Phi' - \Psi)(-0, y)}$$

$$\frac{\varkappa_1}{\mu_1} \Phi(+0, y) - \frac{1}{\mu_1} \overline{(\Phi - z\Phi' - \Psi)(+0, y)} = \frac{\varkappa_2}{\mu_2} \Phi(-0, y) - \frac{1}{\mu_2} \overline{(\Phi - z\Phi' - \Psi)(-0, y)}$$

are satisfied identically if we put [9]

$x \geqslant 0$

$$\Phi(z) = c_1 K(z) + c_3 L(z), \qquad \overline{\Phi}(-z) - z\overline{\Phi}'(-z) - \overline{\Psi}(-z) = c_1 K(z) - c_4 L(z)$$

$x \leqslant 0$

$$\Phi(z) = c_2 K(z) + c_4 L(z), \qquad \overline{\Phi}(-z) - z\overline{\Phi}'(-z) - \overline{\Psi}(-z) = c_2 K(z) - c_3 L(z)$$

Here $K(z)$ and $L(z)$ are holomorphic in the domain S

$$c_1 = (\varkappa_1 + \alpha)^{-1}, \quad c_2 = (1 + \alpha\varkappa_2)^{-1}, \quad c_3 = \alpha(\varkappa_2 + 1), \quad c_4 = \varkappa_1 + 1, \quad \alpha = \mu_1/\mu_2$$

Extracting the singularity $[\Phi, \Psi]^* (z, t)$, and integrating in conformity with (1.3), we obtain the solution of the problem (1.2) as

$$K(z) = \frac{c_4}{2\pi i\Delta} \int_0^1 \left[\frac{q(t)}{c_1(t-z)} + \frac{q(t)}{c_2(t+z)} - \frac{2t\overline{q(t)}}{c_2(t+z)^2} \right] dt$$

$$L(z) = \frac{1}{2\pi i\Delta} \int_0^1 \left[\frac{q(t)}{t-z} - \frac{q(t)}{t+z} + \frac{2t\overline{q(t)}}{(t+z)^2} \right] dt \tag{1.4}$$

The stresses in the solution (1.4) are bounded everywhere including the points O and O_1 (since $q(0) = q(1) = 0$), and vanish at infinity.

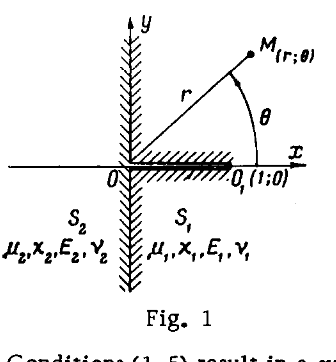

Fig. 1

Without limiting the generality, we can put $q(t)=0$ in (1.1) and can consider two problems with the following boundary conditions:

$$[Y_y]_+ (t) = [Y_y]_- (t) = \operatorname{Re} s(t)$$
$$[X_y]_+ (t) = [X_y]_- (t) = 0 \tag{1.5}$$

$$[Y_y]_+ (t) = [Y_y]_- (t) = 0$$
$$[X_y]_+ (t) = [X_y]_- (t) = \operatorname{Im} s(t) \tag{1.6}$$

Conditions (1.5) result in a symmetric, and (1.6) a skew-symmetric stress-strain state of the system relative to the x-axis; the corresponding problems are henceforth called symmetric and skew-symmetric.

Let us seek the solution of problems belonging to the class of stress-strain states in which the displacements are bounded everywhere, but the stresses are bounded everywhere except the points O, O_1 where they have polar singularities (with exponent $\frac{1}{2}$ at the point O_1, and exponent dependent on the relationships between the elastic constants [10] at the point O), and equal zero at infinity. Because the principal vector of the stress resultants applied to the slit edges is zero, and the displacements are unique, estimates of the form

$$u + iv = O(z^{-1}),\ X + iY = (X + iY)_0 + O(z^{-1}),\ X_x,\ X_y,\ Y_y = O(z^{-2}) \tag{1.7}$$

hold for an arbitrary point $M(z)$ ($|z| > 1$), where u, v are displacements of the point $M(z)$, $X + iY$ is the principal vector of the stress resultants on an arbitrary arc OM, $(X + iY)_0$ is a constant.

2. Reduction of fundamental problems to a Wiener-Hopf functional equation. Let us introduce polar coordinates r, θ (Fig. 1) and associated quantities into the considerations, the stresses σ_r, $\tau_{r\theta}$, σ_θ; the displacements u_r, u_θ, and also functions R, Θ of the form

$$R + i\Theta = -(X + iY)_0 \exp(-i\theta) + \int_0^r (\tau_{r\theta} + i\sigma_\theta)\, dr$$

In conformity with (1.7), for $r > 1$ the following relationships are satisfied

$$f^{(i)} = O(r^{-1}), \qquad f^{(i)} = (u_r,\ u_\theta,\ R,\ \theta)(r, \Theta) \qquad (i = 1, 2, 3, 4)$$

$$g^{(i)} = O(r^{-2}), \qquad g^{(i)} = \left(\frac{\partial u_r}{\partial r}, \frac{\partial u_\theta}{\partial r}, \tau_{r\theta}, \sigma_\theta, \sigma_r\right) \qquad (i = 1, 2, 3, 4, 5)$$

$$g^{(i)} = \frac{\partial f^{(i)}}{\partial r} \qquad (i = 1, \ldots, 4), \qquad g^{(5)} = \nu g^{(4)} + E g^{(1)} \tag{2.1}$$

The quantities $f^{(i)}$ are bounded everywhere, and the quantities $g^{(i)}$ are bounded everywhere except the points O, O_1 where they have polar singularities. Hence, Mellin transformations of the form

$$(\bar{f}, \bar{g})^{(i)}(\theta, p) = \int_0^\infty (f, g)^{(i)}(r, \theta)\, r^{p-1} dr; \quad \operatorname{Re} p = \beta,\ \operatorname{Im} p = \lambda \tag{2.2}$$

$$\bar{g}^{(i)}(\theta, p) = -(p - 1)\bar{f}^{(i)}(\theta, p - 1) \quad (i = 1, \ldots, 4)$$

$$\bar{g}^{(5)}(\theta, p) = (\nu \bar{g}^{(4)} + E\bar{g}^{(1)})(\theta, p)$$

exist.

The functions $\bar{f}^{(i)}$ $(i = 1, \ldots, 4)$ are hence holomorphic in the strip $0 < \beta < 1$, and $\bar{g}^{(i)}$ $(i = 1, \ldots, 5)$ in the strip $1 \leqslant \beta < 2$ in conformity with (2.1).

All the mentioned transformations can be represented as ([1], p. 24)

$$(\bar{f}, \bar{g})^{(i)}(\theta, p) = (\bar{f}, \bar{g})_+^{(i)}(\theta, p) + (\bar{f}, \bar{g})_-^{(i)}(\theta, p) \tag{2.3}$$

$$(\bar{f}, \bar{g})_+^{(i)}(\theta, p) = \int_0^1 (f, g)^{(i)}(r, \theta)\, r^{p-1} dr,\ (\bar{f}, \bar{g})_-^{(i)}(\theta, p) = \int_1^\infty (f, g)^{(i)}(r, \theta)^{p-1} r dr$$

where the $\bar{f}_\pm^{(i)}(\theta, p)$ are holomorphic, respectively, in the half-planes $\beta > 0$, $\beta < 1$, and the $\bar{g}_\pm^{(i)}(\theta, p)$ in the half-planes $\beta > 1$, $\beta < 2$, respectively.

The conditions

$$\bar{f}^{(3)}(0, p) = \bar{f}^{(3)}(\pi, p) = \bar{f}^{(2)}(\pi, p) = 0, \qquad \bar{f}^{(i)}(\pi/2 - 0, p) = \bar{f}^{(i)}(\pi/2 + 0, p) \quad (2.4)$$

are satisfied in the symmetric problem.

It is easy to represent the general solution of the problem in terms of an as yet unknown transformation $\bar{f}^{(2)}(0, p)$. The component $\bar{f}^{(4)}(\theta, p)$ say, is given by Formulas

$$0 \leqslant \theta \leqslant {}^1/_2\pi \quad (2.5)$$

$$\frac{(\varkappa_1 + 1)\sin p\pi \bar{f}^{(4)}(\theta, p)}{2\mu_1 \bar{f}^{(2)}(0, p)} = (p-1)\cos[(p+1)(\pi-\theta)] - (p+1)\cos[(p-1)(\pi-\theta)] +$$

$$+ (\alpha - 1) c_1 (2p+1)[(p-1)\cos(p+1)\theta + p\cos(p-1)\theta] - (\alpha\varkappa_2 - \varkappa_1) c_2 \cos(p-1)\theta$$

$${}^1/_2\pi \leqslant \theta \leqslant \pi$$

$$\frac{\sin p\pi \bar{f}^{(4)}(\theta, p)}{2\mu_1 \bar{f}^{(2)}(0, p)} = c_2 (p-1)\cos[(p+1)(\pi-\theta)] + [c_2 p - c_1(2p+1)]\cos[(p-1)(\pi-\theta)]$$

where the expressions for the remaining components $\bar{f}^{(i)}(\theta, p)$ $(i = 1, 2, 3)$ are completely analogous.

The conditions

$$\bar{f}^{(4)}(0, p) = \bar{f}^{(4)}(\pi, p) = \bar{f}^{(1)}(\pi, p) = 0, \qquad \bar{f}^{(i)}(\pi/2 - 0, p) = \bar{f}^{(i)}(\pi/2 + 0, p) \quad (2.6)$$

are satisfied in the skew-symmetric problem.

The component $\bar{f}^{(3)}(\theta, p)$ is given by Formulas

$$0 \leqslant \theta \leqslant {}^1/_2\pi \quad (2.7)$$

$$\frac{(\varkappa_1 + 1)\sin p\pi \bar{f}^{(3)}(\theta, p)}{2\mu_1 \bar{f}^{(1)}(0, p)} = -(p+1)\cos[(p+1)(\pi-\theta)] + (p-1)\cos[(p-1)(\pi-\theta)] +$$

$$+ (\alpha - 1) c_1 (2p-1)[(p+1)\cos(p+1)\theta + p\cos(p-1)\theta] - (\alpha\varkappa_2 - \varkappa_1) c_2 \cos(p-1)\theta$$

$${}^1/_2\pi \leqslant \theta \leqslant \pi$$

$$\frac{\sin p\pi \bar{f}^{(3)}(\theta, p)}{2\mu_1 \bar{f}^{(1)}(0, p)} = -c_2 (p+1)\cos[(p+1)(\pi-\theta]) -$$

$$- [c_2 p - c_1(2p-1)]\cos[(p-1)(\pi-\theta)]$$

and are completely analogous to the expressions for the other components $\bar{f}^{(i)}(\theta, p)$ $(i = 1, 2, 4)$ where the transformant $\bar{f}^{(1)}(0, p)$ is to be sought.

The boundary conditions of the symmetric problem are given by the equalities

$$\bar{f}^{(2)}(0, p) = \bar{f}_+^{(2)}(0, p), \qquad \bar{f}^{(4)}(0, p) = \bar{f}_-^{(4)}(0, p) + \bar{f}_+^{(4)}(p) \quad (2.8)$$

$$\bar{f}_+^{(4)}(p) = -\frac{Y_0}{p} + \int_0^1 r^{p-1} dr \int_0^r \operatorname{Re} s(t)\, dt$$

in conjunction with (2.4).

The boundary conditions of the skew-symmetric problem are given by

$$\bar{f}^{(1)}(0, p) = \bar{f}_+^{(1)}(0, p), \qquad \bar{f}^{(3)}(0, p) = \bar{f}_-^{(3)}(0, p) + \bar{f}_+^{(3)}(p) \quad (2.9)$$

$$\bar{f}_+^{(3)}(p) = -\frac{X_0}{p} + \int_0^1 r^{p-1} dr \int_0^r \operatorname{Im} s(t)\, dt$$

in combination with (2.6).

Substituting (2.5) into (2.8) and (2.7) into (2.9) yields the same equality

$$H(p)\,P_+(p) - Q_-(p) = F_+(p)\,(0<\beta<1), \qquad H(p) = \frac{4\mu_1 h(p)}{(\varkappa_1+1)\sin p\pi} \tag{2.10}$$

$$h(p) = \cos p\pi - dp^2 - b, \qquad d = -\frac{2(\alpha-1)}{\varkappa_1+\alpha}, \qquad b = \frac{1}{2}\left(\frac{\alpha-1}{\varkappa_1+\alpha} + \frac{\alpha\varkappa_2-\varkappa_1}{1+\alpha\varkappa_2}\right)$$

$$P_+(p) = (\bar f^{(2)}, \bar f^{(1)})(0, p), \qquad Q_-(p) = (\bar f^{(4)}, \bar f^{(3)})_-(0, p), \quad F_+(p) = (\bar f^{(4)}, \bar f^{(3)})_+(p)$$

which is a Wiener-Hopf functional equation to seek the unknown transformants $P_+(p)$, $Q_-(p)$, where an additional condition is needed to determine the constant $(Y, X)_0$ in the expression $F_+(p)$.

The stresses $(\sigma_0, \tau_{r0})(r, 0)$ in the appropriate problems are bounded everywhere except at the point $r=1$ in whose neighborhood there are singularities of the form $(N, T)(r-1)^{-1/2}$ (N, T are coefficients of the normal and tangential stress intensity, respectively), therefore, the principal parts of the functions $(g^{(4)}, g^{(3)})(r, 0)$ are representable as $(N, T)(\ln r)^{-1/2}$ for $r>1$. Hence, for large $|p|$ in the regularity half-plane ($\beta<2$) of the transformant $(\bar g_-^{(4)}(0,p)\ \bar g_-^{(3)})(0, p)$ we obtain the estimates

$$\bar g_-^{(4)}(0, p) = \int_{\ln r=0}^{\infty} g^{(4)}(r, 0)\exp(p\ln r)\,d\ln r = \frac{N\Gamma(1/2)}{i}\,p^{-1/2} + O(p^{-1})$$

$$\bar g_-^{(3)}(0, p) = \int_{\ln r=0}^{\infty} g^{(3)}(r, 0)\exp(p\ln r)\,d\ln r = \frac{T\Gamma(1/2)}{}\,p^{-1/2} + O(p^{-1}) \tag{2.11}$$

By virtue of the boundedness of the functions $(g^{(4)}, g^{(3)})(r, 0)$ in the interval [0, 1] analogous estimates hold for the transformants $(\bar g^{(4)}, \bar g^{(3)})(0, p)$ in the strip $1 \leqslant \beta < 2$. In conformity with (2.2), an estimate of the form $0\,(p^{-1/2})$ is valid for the transformant $(\bar f^{(4)}, \bar f^{(3)})(0, p)$ in the strip $0<\beta<1$, and by virtue of the boundedness of $H(p)$ in the mentioned strip, the last estimate also holds for the transformants $(\bar f^{(2)}, \bar f^{(1)})(0, p)$. We obtain

$$\bar f^{(i)}(0, p) = O(p^{-1/2}) \qquad (0<\beta<1, \ i = 1, \ldots, 4) \tag{2.12}$$

Let us note an important particular case. For $0 \leqslant r \leqslant 1$ let

$$(g^{(4)}, g^{(3)})_m(r, 0) = r^m \qquad (m = 0, 1, \ldots) \tag{2.13}$$

We have then

$$F_+(p) = -\frac{(Y, X)_0}{p} + \frac{1}{(m+1)(p+m+1)} \tag{2.14}$$

3. Factorization of the function $H(p)$ and solution of (2.10). Let $p_* = \beta_* + i\lambda_*$ be the major root, in absolute value, of the function $h(p)$

$$\cos\beta_*\pi\ \mathrm{ch}\ \lambda_*\pi = d(\beta_*^2 - \lambda_*^2) - b, \quad \sin\beta_*\pi\ \mathrm{sh}\ \lambda_*\pi = -2d\beta_*\lambda_* \tag{3.1}$$

An appropriate analysis shows that the equalities (3.1) can be satisfied if relationships of the following kind (k is an integer)

$$p_* = 2k + iO(\ln k) + O(k^{-1}\ln k) \qquad (d>0)$$

$$p_* = 2k - 1 + iO(\ln k) + O(k^{-1}\ln k) \qquad (d<0)$$

are satisfied.

The quantities $\bar p_*, -p_*, -\bar p_*$ will also be the desired roots (the latter two because of the evenness of $h(p)$).

Near the origin, the function $h(p)$ has different real roots, in particular, whose value may be arbitrarily large for sufficiently small values of the quantity $1-\alpha$. However, direct calculations easily verify that there exist $2k+1$ roots of the function $h(p)$ for values of the constant α in the interval $(0 \leqslant \alpha < 1)$ in the strip $0 < \beta < 2k + {}^1/_2$, and there are $2k$ roots of the same function for values of the constant $1/\alpha$ in the range $(0 \leqslant 1/\alpha < 1)$. Hence, a countable set of roots with positive abscissas can be enumerated as follows: $p_0, p_1, p_2, \ldots, p_{2k-1}, p_{2k}, \ldots$ $(d > 0)$; $p_1, p_2, \ldots, p_{2k-1}, p_{2k}, \ldots$ $\ldots$ $(d < 0)$.

The first root $p_{0,1}$ is always a real number belonging to the interval [0, 1], and for large k formulas of the following kind hold

$$\begin{aligned} p_{2k-1} &= 2k + iO(\ln k) + O(k^{-1}\ln k), \quad p_{2k} = \bar{p}_{2k-1} \quad (d > 0) \\ p_{2k-1} &= 2k - 1 + iO(\ln k) + O(k^{-1}\ln k), \quad p_{2k} = \bar{p}_{2k-1} \quad (d < 0) \end{aligned} \tag{3.2}$$

Roots with negative abscissas can be enumerated analogously: $p_{-0} = -p_0$ $(d > 0)$, $p_{-2k+1} = -p_{2k-1}$, $p_{-2k} = -p_{2k}$ $(k = 1, 2, \ldots)$

As has been shown in [11], by virtue of (3.2) infinite products of the form

$$\prod_{k=1}^{\infty}\left(1 + \frac{p}{p_{2k-1}}\right)\left(1 + \frac{p}{p_{2k}}\right)\left(1 + \frac{p}{2k}\right)^{-2} \qquad (d > 0)$$

$$\prod_{k=1}^{\infty}\left(1 + \frac{p}{p_{2k-1}}\right)\left(1 + \frac{p}{p_{2k}}\right)\left(1 + \frac{p}{2k-1}\right)^{-2} \qquad (d < 0)$$

converge absolutely and uniformly in the half-planes $\beta > -p_{0,1}$. The considered infinite products hence tend to finite limits when $|p| \to \infty$ in the corresponding half-planes. Taking account of Formula $[\Gamma(p)]^{-1} = pe^{Cp}\prod_{k}\left(1 + \frac{p}{k}\right)\exp\frac{-p}{k}$

(C is the Euler-Mascheroni constant), and the resulting identities

$$\prod_{k=1}^{\infty}\left(1 + \frac{p}{2k}\right)\exp\frac{-p}{2k} = 2e^{-Cp/2}[p\Gamma({}^1/_2 p)]^{-1}$$

$$\prod_{k=1}^{\infty}\left(1 + \frac{p}{2k-1}\right)\exp\frac{-p}{2k-1} = e^{-Cp/2}\,\Gamma({}^1/_2 p)\,[2\Gamma(p)]^{-1}$$

we arrive at the conclusion that infinite products of the form

$$\Pi_1(p) = \prod_{k=1}^{\infty}\left(1 + \frac{p}{p_{2k-1}}\right)\left(1 + \frac{p}{p_{2k}}\right)\exp\frac{-p}{k} \qquad (d > 0)$$

$$\Pi_2(p) = \prod_{k=1}^{\infty}\left(1 + \frac{p}{p_{2k-1}}\right)\left(1 + \frac{p}{p_{2k}}\right)\exp\frac{-2p}{2k-1} \qquad (d < 0)$$

converge absolutely and uniformly in the considered half-planes, and for $|p| \to \infty$ the following estimates hold:

$$\begin{aligned} \Pi_1(p) &= e^{-Cp}[p\Gamma({}^1/_2 p)]^{-2}\,O(1) \qquad (d > 0) \\ \Pi_2(p) &= e^{-Cp}[\Gamma({}^1/_2 p)]^2[\Gamma(p)]^{-2}\,O(1) \qquad (d < 0) \end{aligned} \tag{3.3}$$

We factorize the function $h(p)$ as follows:

$$h(p) = h_+(p)\, h_-(p), \qquad h_-(p) = h_+(-p)$$

$$h_+(p) = \begin{cases} (1-b)^{1/2}\, 2^{-p} e^{Cp} (p_0 + p / p_0)\, \Pi_1(p) & (d > 0) \\ (1-b)^{1/2}\, 2^{p} e^{Cp}\, \Pi_2(p) & (d < 0) \end{cases} \tag{3.4}$$

Taking account of the known formula $\pi / \sin p\pi = \Gamma(p)\,\Gamma(1-p)$, we furthermore factorize the function $H(p)$

$$H(p) = \frac{H_+(p)}{H_-(p)}, \qquad H_-(p) = -\frac{1}{pH_+(-p)} \tag{3.5}$$

$$H_+(p) = \left[\frac{4\mu_1}{\pi(\varkappa_1 + 1)}\right]^{1/2} \Gamma(p)\, h_+(p), \qquad H_-(p) = \left\{\left[\frac{4\mu_1}{\pi(\varkappa_1+1)}\right]^{1/2} \Gamma(1-p)\, h_-(p)\right\}^{-1}$$

where $H_+(p)$ is regular for $\beta > 0$, and $H_-(p)$ for $\beta < p_{0,1}$. For large $|p|$ estimates resulting directly from (3.3) – (3.5) hold

$$H_+(p) = \frac{2^{-p} e^{Cp} (1 + p / p_0)\, e^{-Cp}\, \Gamma(p)}{[p\Gamma(p/2)]^2} O(1) = \frac{2^{-p}\,\Gamma(p)}{p\,[\Gamma(p/2)]^2} O(1) \qquad (d > 0)$$

$$H_-(p) = \frac{2^{p} e^{Cp}\, [\Gamma(p/2)]^2 e^{-Cp}\, \Gamma(p)}{[\Gamma(p)]^2} O(1) = \frac{2^{p}\,[\Gamma(p/2)]^2}{\Gamma(p)} O(1) \qquad (d < 0)$$

Utilizing formulas of doubling the gamma function, we have

$$H_+(p) = \frac{\Gamma(p/2 + 1/2)}{p\Gamma(p/2)} O(1) \quad (d > 0), \qquad H_+(p) = \frac{\Gamma(p/2)}{\Gamma(p/2 + 1/2)} O(1) \quad (d < 0)$$

Applying the Stirling formula, we obtain

$$\Gamma(p/2 + 1/2) \sim (p/2)^{1/2}\, \Gamma(p/2), \quad H_+(p) \sim p^{-1/2}\, [G + o(1)] \tag{3.6}$$

$$H_-(p) \sim p^{-1/2}\, [(iG)^{-1} + o(1)]$$

where G is a constant to be determined. By virtue of (3.5), (3.6), in the strip $0 < \beta < p_{0,1}$ of regularity of the functions $H(p)$, $H_\pm(p)$ we have for large $|p|$

$$H(p) = i\, G^2 + o(1) \tag{3.7}$$

From (2.10) and (3.7) we obtain the value $G = [4\mu_1 / (\varkappa_1 + 1)]^{1/2}$ for the constant G, and we arrive at the following asymptotic formulas:

$$(|p| \to \infty)\; H_+(p) \sim \left(\frac{4\mu_1}{\varkappa_1 + 1}\right)^{1/2} p^{-1/2} \qquad H_-(p) \sim -i \left(\frac{4\mu_1}{\varkappa_1 + 1}\right)^{-1/2} p^{-1/2} \tag{3.8}$$

Let us turn to the solution of the functional equation (2.10). In the strip $0 < \beta < p_{0,1}$ we have

$$H_+(p)\; P_+(p) \;-\; H_-(p)\; Q_-(p) \;=\; H_-(p)\; F_+(p) \tag{3.9}$$

Taking account of (2.8), (2.9) and (3.8), we obtain the estimate $O(p^{-3/2})$ for $H_-(p)\; F_+(p)$. In any interior strip $0 < \beta_- \leqslant \beta \leqslant \beta_+ < p_{0,1}$ we have [1]

$$H_-(p)\; F_+(p) = E_+(p) + E_-(p) \tag{3.10}$$

Here the $E_\pm(p)$ are regular in the half-planes $\beta > \beta_-$ and $\beta \leqslant \beta_+$, respectively. We obtain ([1], p. 49)

$$H_+(p)\; P_+(p) - E_+(p) = J(p), \quad H_-(p)\; Q_-(p) + E_-(p) = J(p) \tag{3.11}$$

where $J(p)$ is an analytic function in the whole p plane. Let us assume compliance with the condition

$$E_\pm(p) = o(1) \;\; (|p| \to \infty) \tag{3.12}$$

Then $J(p) \equiv 0$ by virtue of (2.20) and (3.8), and, moreover, we have according to (3.11)

$$H_+(p)\ P_+(p) - E_+(p) = 0 \qquad (3.13)$$

$$H_-(p)\ Q_-(p) + E_-(p) = 0 \qquad (3.14)$$

Combining (3.10) and (3.14), we obtain relationship

$$H_-(p)\ (\bar{f}^{(4)}, \bar{f}^{(3)})\ (0,\ p) - E_+(p) = 0 \qquad (3.15)$$

since $F_+(p) + Q_-(p) = (\bar{f}^{(4)}, \bar{f}^{(3)})\ (0,\ p)$ in conformity with (2.10). Therefore, taking account of (3.13) and (3.15) the condition (3.12) results in the following solution of the problem:

$$(\bar{f}^{(2)}, \bar{f}^{(1)})(0, p) = \frac{E_+(p)}{H_+(p)}, \qquad (\bar{f}^{(4)}, f^{(3)})(0, p) = \frac{E_+(p)}{H_-(p)} \quad (3.16)$$

Let us now consider the particular case of a power loading with integer exponent defined by (2.13), (2.14) on the slit. We have

$$E_+(p) = -\frac{H_-(0)(Y, X)_0}{p} + \frac{H_-(-m-1)}{(m+1)(p+m+1)} \qquad (3.17)$$

$$E_-(p) = H_-(p)\left[-\frac{(Y, X)_0}{p} + \frac{1}{(m+1)(p+m+1)}\right] - E_+(p)$$

Hence, there results directly that (3.12) is satisfied and Eqs. (3.16) are true.

In conformity with (2.12), the left sides of (3.16) have order $O(p^2)$ for $|p| \to \infty$, and the right sides should have the same order, which taking account of (3.8) yields

$$-H_-(0)(Y, X)_0 + \frac{H_-(-m-1)}{m+1} = 0 \qquad (3.18)$$

It is expedient to eliminate the constants $(Y, X)_0$ from (3.17), (3.18); we hence obtain finally

$$E_+(p) = -\frac{H_-(-m-1)}{p(p+m+1)} \qquad (3.19)$$

$$(\bar{f}^{(2)}, \bar{f}^{(1)})(0, p) = -\frac{H_-(-m-1)}{p(p+m+1)H_+(p)}, \qquad (\bar{f}^{(4)}, \bar{f}^{(3)})(0, p) = -\frac{H_-(-m-1)}{p(p+m+1)H_-(p)}$$

Taking account of the infinite product representation of $[\Gamma(p)]^{-1}$, and of the identity

$$\sum_{k=1}^{\infty}\left(\frac{1}{2k-1} - \frac{1}{2k}\right) = \ln 2$$

the functions $H_\pm(p)$ can be given the following form which is more convenient for practical calculations:

$$H_+(p) = \left[\frac{4\mu_1(1-b)}{\pi(\varkappa_1+1)}\right]^{1/2} \Pi_+(p), \qquad H_-(p) = -\left[\frac{4\mu_1(1-b)}{\pi(\varkappa_1+1)}\right]^{-1/2} \frac{1}{p\Pi_+(-p)} \quad (3.20)$$

$$\Pi_+(p) = \prod^{\infty}\left(1 + \frac{p}{p_{2k-1}}\right)\left(1 + \frac{p}{p_{2k}}\right)\left(1 + \frac{p}{2k-1}\right)^{-1}\left(1 + \frac{p}{2k}\right)^{-1} \begin{cases}(p_0+p)/p_0 p & (d>0)\\ 1/p & (d<0)\end{cases}$$

Let us note the limiting case $\mu_2/\mu_1 = 1/\alpha = 0$ (free right half-plane with a notch). We have

$$H(p) = \frac{4\mu_1 h(x)}{(\varkappa_1+1)\sin p\pi}, \qquad h(p) = \cos p\pi + 2p^2 - 1, \qquad p_1 = p_{-1} = 0,\ p_2 = 1$$

Now the function $H(p)$ does not have a pole at the origin, as before, but a radical, and the factorization formulas change correspondingly

$$H(p)=\frac{H_+(p)}{H_-(p)},\qquad H_-(p)=\frac{p}{H_+(-p)},\qquad H_+(p)=\left[\frac{4\mu_1}{\pi(\varkappa_1+1)}\right]^{1/2}\Gamma(p)\,h_+(p)$$

$$h_+(p)=(\pi^2-4)^{1/2}\,2^{p-1/2}\,e^{(C-2)}\;p^2(1+p)\prod_{k=2}^{\infty}\left(1+\frac{p}{p_{2k-1}}\right)\left(1+\frac{p}{p_{2k}}\right)e^{\frac{-p}{k}}\tag{3.21}$$

For large $|p|$ the following asymptotic relationships are valid

$$H_+(p)\sim\left(\frac{4\mu_1}{\varkappa_1+1}\right)^{1/2}p^{1/2},\qquad H_-(p)\sim -i\left(\frac{4\mu_1}{\varkappa_1+1}\right)^{-1/2}p^{1/2}\tag{3.22}$$

If condition (3.12) is satisfied, then (3.16) remain valid for the general loading case, where the function $F_+(p)$ is defined by (2.8), (2.9) in which we should put $(Y, X)_0 = 0$. For the particular case of an r^m loading (3.16) becomes

$$E_+(p)=\frac{H_-(-m-1)}{(m+1)(p+m+1)},\qquad (\bar{j}^{(2)},\bar{j}^{(1)})(0,p)=\frac{H_-(-m-1)}{(m+1)(p+m+1)H_+(p)}$$

$$(\bar{j}^{(4)},\bar{j}^{(3)})(0,p)=\frac{H_-(-m-1)}{(m+1)(p+m+1)H_-(p)}\tag{3.23}$$

where the solution (3.23) satisfies condition (2.12) by virtue of (3.22).

4. Stress concentration near the ends of the slit. The transformants $\bar{f}^{(i)}(0, p)$ satisfy condition (2.12) in the obtained solutions for power law loadings (3.19), (3.22), and the corresponding functions remain bounded for $\theta=0$ $(0\leqslant r<\infty)$, the transformants $\bar{g}^{(i)}(0, p)$ are of the order $O(p^{-1/2})$ for large $|p|$, and hence, the corresponding functions have a polar singularity at the point O_1 with a polarity exponent $1/2$, as has been specified earlier (Section 1). To determine the stress intensity coefficients in the neighborhood of the point O_1 $(N, T)_m$ in the presence of the loading (2.13), according to (2.2), (2.11), (3.19), (3.23), we obtain in the general and limit cases

$$-(N, T)_m=\left[\frac{4\mu_1}{\pi(\varkappa_1+1)}\right]^{1/2}H_-(-m-1)=\frac{1}{(m+1)\Gamma(m+1)h_+(m+1)}\quad\left(\frac{1}{\alpha}\neq 0\right)\tag{4.1}$$

$$-(N, T)_m=-\left[\frac{4\mu_1}{\pi(\varkappa_1+1)}\right]^{1/2}\frac{H_-(-m-1)}{m+1}=\frac{1}{\Gamma(m+1)h_+(m+1)}\qquad\left(\frac{1}{\alpha}=0\right)$$

Taking account of (3.8), (3.22), the following asymptotic formulas result from (4.1):

$$(N,\ T)_m\sim[\pi(m+1)]^{-1/2}\quad(m\gg 1)\tag{4.2}$$

Values of the intensity coefficients $(N, T)_m^{(l)}$ for a slit of length l are $(N, T)_m l^{m+1/2}$. Let the loading on a slit of length l be given as a power series

$$(g^{(4)}, g^{(3)})(r, 0)=\sum_{m=0}^{\infty}a_m r^m,\qquad \rho=\lim|a_m|^{-1/m}>l\qquad(0\leqslant r\leqslant l)\tag{4.3}$$

where ρ is the radius of convergence.

Because of (4.2), multiplying the coefficients a_m by the quantity $(N,\ T)_m$ keeps the quantity ρ unchanged, and hence the following series converges

$$(N, T)=\sum_{m=0}^{\infty}a_m(N, T)_m\,l^{m+1/2}\tag{4.4}$$

which in this case determines the values of the stress intensity coefficients (the series (4.3), (4.4) can be some polynomials, in particular).

It follows from (4.4) that in the symmetric problem the length of an equilibrium crack originated in the main stress field (4.3) and propagated into the right half-plane S_1, should satisfy a relationship such as [12 – 14]

$$l^{1/2} \sum_{m=0}^{\infty} a_m N_m l^m \leqslant \frac{K}{\pi} \tag{4.5}$$

where K is the modulus of adhesion of the material in the right half-plane.

Turning to a representation of the function $(f, g)^{(i)}(r, \theta)$ $(i = 1, \ldots, 4)$ in the closed intervals $0 < \varepsilon \leqslant r \leqslant r_1 < 1$, $1 < r_2 \leqslant r \leqslant \varepsilon^{-1} < \infty$ as power series, we, as usual, supplement the contour of integration in the inversion formulas for

$$(f, g)^{(i)}(r, \theta) = \frac{1}{2\pi i} \int_{\beta_*-i\infty}^{\beta_*+i\infty} (\bar{f}, \bar{g})^{(i)}(\theta, p)\, r^{-p}\, dp \qquad (i = 1, \ldots, 4)$$
$$\bar{f}^{(i)}(\theta, p) \quad (0 < \beta_* < 1), \qquad \bar{g}^{(i)}(\theta, p) \quad (1 < \beta_* < 2) \tag{4.6}$$

by semicircles $|p| = \text{const}$ in the half-planes $\beta < \beta_*$ and $\beta > \beta_*$ for the first and second intervals, respectively. It follows from (2.5) and (2.7) that the transformants $\bar{f}^{(i)}(\theta, p)$ $(i = 1, \ldots, 4)$, $(\theta \neq 0)$ are representable as

$$\bar{f}^{(i)}(\theta, p) = (\bar{f}^{(2)}, \bar{f}^{(1)})(0, p) \csc p\pi \sum_j (c_{j1}^{(i)} p + c_{j0}^{(i)}) (\sin, \cos) [(p \pm 1) \varphi]$$
$$0 < \varphi = (\theta, \pi - \theta) \leqslant \pi/2), \quad 0 \leqslant \varphi = \pi - \theta \leqslant \pi/2$$
$$(0 < \theta \leqslant \pi/2, \quad (\pi/2 \leqslant \theta \leqslant \pi) \tag{4.7}$$

where the $(c_{j1}, c_{j0})^{(i)}$ are constants connected with the elastic constants of the half-planes $S_{1,2}$. Proceeding from (4.7) we can obtain the following estimates for the integrals over the semicircular supplementing contours (4.6) $(\theta \neq 0)$

$$|\pi p r^{-p} (\bar{f}, \bar{g})^{(i)}(\theta, p)| \sim (|p|^{1/2}, |p|^{3/2}) \exp[-|p| \min(|\ln r_{1,2}|; \theta)] = o(1)$$

i.e. the mentioned integrals tend to zero uniformly in r in the considered intervals as $|p|$ grows; for $\theta = 0$ an analogous result follows directly from the Jordan lemma since

$$(\bar{f}, \bar{g})^{(i)}(\theta, p) = O(p^{-3/2}, p^{-1/2}) \qquad (i = 1, \ldots, 4)$$

Application of residue theorems for (4.6) results in the following equalities:

$$(f, g)^{(i)}(r, \theta) = \sum{}' \operatorname{Res} [r^{-p} (\bar{f}, \bar{g})^{(i)}(\theta, p)] \qquad (0 < \varepsilon < r \leqslant r_1 < 1)$$
$$(f, g)^{(i)}(r, \theta) = -\sum{}'' \operatorname{Res} [r^{-p} (\bar{f}, \bar{g})^{(i)}(\theta, p)] \qquad (1 < r_2 < r < \varepsilon^{-1} < \infty)$$

where Σ' and Σ'' denote uniformly convergent series of residues of the expressions $r^{-p} (\bar{f}, \bar{g})^{(i)}(\theta, p)$ $(i = 1, \ldots, 4)$, extended over the poles of the transformant in the $\beta < \beta_*$ and $\beta > \beta_*$ half-planes, respectively. The expressions $g^{(i)}(r, \theta)$ may be obtained by direct differentiation of the corresponding series $f^{(i)}(r, \theta)$.

There results directly from (4.7) that the poles of the transformant $\bar{f}^{(i)}(\theta, p)$ $(0 < \theta \leqslant \pi,\ i = 1, \ldots, 4)$ are poles of the meromorphic function $\csc p\pi\,[\bar{f}^{(2)}, \bar{f}^{(1)}](0, p)$; it follows from the equalities (3.19), (3.5), (3.23), (3.21) that these poles for the loading (2.13) are roots of the functions

$$\frac{(p+m+1)\,h_+(p)}{\Gamma(-p)} \quad \left(\frac{1}{\alpha} \neq 0\right) \qquad \text{or} \qquad \frac{(p+m+1)\,h_+(p)}{\Gamma(1-p)} \quad \left(\frac{1}{\alpha} = 0\right)$$

Therefore, in the interval $1 < r_2 \leqslant r \leqslant \varepsilon^{-1} < \infty$, Laurent expansions hold $(i = 1, \ldots, 4)$

$$f^{(i)}(r, \theta) = -\sum_{k=1}^{\infty} r^{-k} \operatorname{Res} \bar{f}^{(i)}(\theta, k), \quad g^{(i)}(r, \theta) = \sum_{k=1}^{\infty} r^{-k-1} k \operatorname{Res} \bar{f}^{(i)}(\theta, k)$$

which are valid to the infinitely distant point $(\varepsilon = 0)$ in whose neighborhood $(f, g)^{(i)}(r, \theta) = O(r^{-1}, r^{-2})$ $(i = 1, \ldots 4)$, as has been specified earlier (1.7). In the interval $0 < \varepsilon \leqslant r \leqslant r_1 < 1$ the following expansions hold $(i = 1, \ldots, 4)$

$$f^{(i)}(r, \theta) = r^{m+1} \operatorname{Res} \bar{f}^{(i)}(\theta, -m-1) +$$

$$+ \frac{1 + \operatorname{sign} d}{2} r^{p_0} \operatorname{Res} \bar{f}^{(i)}(\theta, -p_0) + \sum_{k=1}^{\infty} f_k^{(i)}(r, \theta) \tag{4.8}$$

$$f_k^{(i)}(r, \theta) = r^{p_{2k-1}} \operatorname{Res} \bar{f}^{(i)}(\theta, -p_{2k-1}) + r^{p_{2k}} \operatorname{Res} \bar{f}^{(i)}(\theta, -p_{2k})$$

which are valid to the vertex $(\varepsilon = 0)$ in whose neighborhood $f^{(i)}(r, \theta) = O(r^{p_{0,1}})$, i.e. are bounded (Section 1).

Expansions representable as $(1/\alpha \neq 0)$

$$g^{(i)}(r, \theta) = r^{p_{0,1}-1} p_{0,1} \operatorname{Res} \bar{f}^{(i)}(\theta, -p_{0,1}) + g_0^{(i)} r, \theta) \quad (i = 1, \ldots, 4) \tag{4.9}$$

can be obtained for $g^{(i)}(r, \theta)$ by term by term differentiation in (4.8), where the first members are polar singularities at the point O (Section 1) with polar exponent $1 - p_{0,1}$, and the second members are uniformly convergent expansions of bounded functions down to $\varepsilon = 0$. In the limit case $(1/\alpha = 0)$ the $g^{(i)}(r, \theta)$ are regular since $f_1^{(i)}(r, \theta) = \text{const} + r \operatorname{Res} \bar{f}^{(i)}(\theta, -1)$.

Therefore, in all cases the solution of the problem belongs to the considered class of stress-strain states (Section 1).

The stress intensity coefficients $(\bar{N}, \bar{T})_m$ on the continuation of the slit $\theta = \pi$ are given for the loading (2.13) by Formulas

$$(\bar{N}, \bar{T})_m = p_{0,1} \operatorname{Res} (\bar{f}^{(4)}, \bar{f}^{(3)})(\pi, -p_{0,1})$$

Taking account of (2.5), (2.7), (3.19), (3.5) and (4.1), we obtain

$$\bar{N}_m = N_m (m + 1 - p_{0,1})^{-1} A(p_{0,1}) M(p_{0,1}), \quad A(p) = \frac{2p-1}{\varkappa_1 + \alpha} - \frac{2p+1}{1 + \alpha\varkappa_2} \tag{4.10}$$
$$\bar{T}_m = T_m (m + 1 - p_{0,1})^{-1} B(p_{0,1}) M(p_{0,1}),$$

$$B(p) = \frac{2p-1}{1+\alpha\varkappa_2} - \frac{2p+1}{\varkappa_1 + \alpha}, \quad M(p) = \frac{\varkappa_1 + 1}{2} \Gamma(1-p)(p + p_{0,1})[h_+(p)]^{-1}$$

For the loading (4.3) on a slit of length l we have

$$(\overline{N}, \overline{T}) = \sum_{m=0}^{\infty} a_m (\overline{N}, \overline{T})_m \; l^{m^+}$$

$$(m^* = m + 1 - p_{0,1}) \qquad (4.11)$$

where $\overline{N}$, $\overline{T}$ are stress intensity coefficients at $\theta = \pi$.

By virtue of (4.10) and (4.2)

$$(\overline{N}, \overline{T})_m \sim m^{-3/2},$$

and the radius of convergence of (4.3) does not change upon passing to (4.11).

Presented in Table 1 are values of the intensity coefficients N_m for $m = 0,1,2,3$, and also the first roots $p_{0,1}$ and the function $M(p_{0,1})$. According to a special program for the "Ural-2" computer, the real roots were determined first, and then complex roots of the function $h(p)$ with the corresponding value of k by the Newton method

$$p_*^{(s+1)} = p_*^{(s)} + \Delta p^{(s+1)}$$

$$\Delta p^{(s+1)} = -h(p_*^{(s)}) \, [h'(p_*^{(s)})]^{-1}$$

$$(s = 0, 1, \ldots \, s_+)$$

Asymptotic values of the form

$$2k + (i/\pi) \ln 8dk^2 \quad (d > 0)$$

$$2k - 1 + (i/\pi) \ln [2d(2k-1)^2] \quad (d < 0)$$

were taken as the initial approximation $p_*^{(0)}$ in computing the complex roots.

The error in computing the roots did not exceed 0.001, and the total error in determining the desired quantities was 0.01. For $m \geqslant 4$, the asymptotic values of $(N, T)_m$ of the form (4.2), whose difference from the actual values does not exceed 0.02, can be used. In the limit case of a half-plane with a notch ($1/\alpha = 0$) the value $N_0 = 0.788$, presented in Table 1, is in good agreement with the values presented in [4 – 7].

Table 1

$\varkappa$	α	N_m, $m=0$	$m=1$	$m=2$	$m=3$	$p_{0,1}$	$M(p_{0,1})$
	0	0.456	0.355	0.300	0.265	0.631	1.023
	0.20	0.473	0.362	0.305	0.268	0.560	0.969
	0.40	0.483	0.367	0.307	0.270	0.530	0.955
	0.60	0.489	0.369	0.309	0.271	0.514	0.964
	0.80	0.494	0.372	0.310	0.272	0.505	0.979
	1.00	0.500	0.375	0.312	0.273	0.500	1.000
3.00	1.25	0.510	0.378	0.313	0.274	0.474	0.970
	2.50	0.541	0.390	0.320	0.278	0.398	0.976
	5.00	0.576	0.403	0.326	0.283	0.327	1.007
	10.00	0.614	0.417	0.333	0.288	0.258	1.043
	40.00	0.684	0.441	0.346	0.296	0.144	1.096
	∞	0.788	0.475	0.364	0.307		
	0	0.429	0.342	0.293	0.260	0.807	1.147
	0.20	0.451	0.353	0.299	0.265	0.670	0.919
	0.40	0.467	0.360	0.303	0.267	0.601	0.862
	0.60	0.480	0.365	0.307	0.270	0.557	0.841
	0.80	0.490	0.370	0.309	0.271	0.525	0.833
	1.00	0.500	0.375	0.312	0.273	0.500	0.833
2.33	1.25	0.510	0.378	0.313	0.274	0.475	0.812
	2.50	0.543	0.391	0.320	0.279	0.399	0.824
	5.00	0.580	0.405	0.327	0.283	0.323	0.853
	10.00	0.620	0.419	0.335	0.288	0.251	0.880
	40.00	0.689	0.443	0.347	0.296	0.138	0.917
	∞	0.788	0.475	0.364	0.307		
	0.	0.434	0.344	0.294	0.261	0.710	0.721
	0.20	0.453	0.353	0.299	0.265	0.634	0.694
	0.40	0.467	0.359	0.303	0.267	0.584	0.687
	0.60	0.487	0.365	0.306	0.269	0.550	0.677
	0.80	0.490	0.370	0.309	0.271	0.522	0.693
	1.00	0.500	0.375	0.312	0.273	0.500	0.700
1.80	1.25	0.510	0.378	0.313	0.274	0.477	0.688
	2.50	0.544	0.391	0.320	0.279	0.401	0.706
	5.00	0.583	0.405	0.327	0.284	0.321	0.728
	10.00	0.624	0.421	0.335	0.289	0.246	0.748
	40.00	0.693	0.444	0.348	0.297	0.133	0.775
	∞	0.788	0.475	0.364	0.307		
	0	0.433	0.343	0.293	0.260	0.595	0.371
	0.20	0.451	0.351	0.298	0.264	0.574	0.409
	0.40	0.465	0.358	0.302	0.267	0.553	0.440
	0.60	0.478	0.364	0.306	0.269	0.534	0.465
	0.80	0.489	0.369	0.309	0.271	0.516	0.483
	1.00	0.500	0.375	0.312	0.273	0.500	0.500
1.00	1.25	0.511	0.378	0.313	0.274	0.481	0.499
	2.50	0.547	0.392	0.320	0.279	0.408	0.527
	5.00	0.589	0.408	0.329	0.284	0.323	0.540
	10.00	0.635	0.429	0.342	0.293	0.244	0.560
	40.00	0.698	0.446	0.349	0.298	0.128	0.588
	∞	0.788	0.475	0.364	0.307		

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